MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 4 - 2017

Log Functions and Continuous Distributions

Resource Free

ALL SAINTS' COLLEGE

Name: SOLUTIONS **Teacher:** ____________

Marks: 15

Time Allowed: 15 minutes

Instructions: You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1) [1,1,2,2 = 6 marks]

a) Write $\log_2 128 = 7$ in index form.

$2^7 = 128$ ✓

b) Evaluate $\log_6 216$

3 ✓

c) Find $h'(1)$ exactly where $h(x) = 4x\ln e^{2\pi}$

8π ✓✓

d) Evaluate $\dfrac{\log_3 64}{\log_3 0.5}$

$\dfrac{\log_3 2^6}{\log_3 2^{-1}} = \dfrac{6}{-1} = -6$ ✓✓

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2) [3,3 = 6 marks]

The continuous random variable Z is defined by the probability density function

$$f(z)=\begin{cases}\dfrac{t}{z} & 1\le z\le 3\\ 0 & elsewhere\end{cases}$$

(a) Determine the exact value of t

$\int_1^3 \frac{t}{z}\,dz = 1 \Rightarrow [t\ln z]_1^3 = 1$ ✓

$\Rightarrow t\ln 3 - t\ln 1 = 1$ ✓

$\therefore t = \frac{1}{\ln 3}$ ✓

(b) Determine the exact value of $P(1 < Z < 2)$

$\frac{1}{\ln 3}\int_1^2 \frac{1}{z}\,dz = \frac{1}{\ln 3}[\ln z]_1^2$ ✓

$= \frac{1}{\ln 3}[\ln 2 - \ln 1]$ ✓

$= \frac{\ln 2}{\ln 3}$ ✓

3) [3 marks]

A research product determines that the average number of rabbits living on farms throughout Western Australia is 85 per farm with a variance of 49. Given that the distribution of rabbits is normally distributed what is the probability that a randomly chosen farm has between 71 and 92 rabbits?

$P(-2 < X < 1)$

$\approx 13.5\% + 34\% + 34\%$

$\approx 81.5\%$ or 0.815 ✓✓✓

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MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 4 - 2017

Log Functions and Continuous Distributions

Resource Rich

Name: ______________________ **Teacher:** ____________

Marks: 35

Time Allowed: 30 minutes

Instructions: You are allowed a ClassPad and scientific calculator but NO notes.

You will be supplied with a formula sheet.

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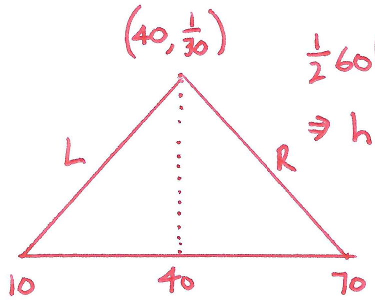
1. [5,1,4 = 10 marks]

A continuous random variable X defined on the interval [10,70] has a symmetrical triangular probability density function.

a) Find the probability density function $p(x)$.



$\frac{1}{2}60h = 1 \Rightarrow h = \frac{1}{30}$

$m_L = \frac{y_2-y_1}{x_2-x_1} = \frac{1}{900}$, hence $m_R = -\frac{1}{900}$ ✓

$$\therefore \; p(x) = \begin{cases} \frac{1}{900}(x-10) & \text{for } 10 \le x \le 40 \checkmark\checkmark \\ -\frac{1}{900}(x-70) & \text{for } 40 \le x \le 70 \checkmark\checkmark \end{cases}$$

b) Find E(X)

40 ✓

c) Find Var[X] and also σ_X

$$Var[X] = \int_{10}^{40} \frac{1}{900}(x-10)(x-40)^2\,dx + \int_{40}^{70} -\frac{1}{900}(x-70)(x-40)^2\,dx \quad \checkmark\checkmark$$

$= \underline{150}$ ✓ hence $\sigma_x = \sqrt{150} \approx 12.247$ ✓

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2) [1,2,3 = 6 marks]

A patient is given a dose of an experimental drug. Sometime later a second dose of the same experimental drug is administered. The effective amount x units of the drug in the patient's bloodstream t minutes from administering the second dose is modelled by:

$$x = \ln(3t + e^2) \quad where \quad t \geq 0$$

a) How much of the drug is present in the bloodstream at the instant the second dose is given?

2 units ✓

b) How much of the drug is present exactly five hours after the second dose is given?

When $t = 300$

$x = \ln(900 + e^2)$ ✓✓

$= 2\ln 900 \approx 6.8106$ units

c) Find the rate of change of the amount of drug in the bloodstream after 4 hours correct to three decimal places.

$\frac{dx}{dt} = \frac{3}{3t + e^2}$ ✓

When $t = 240$ ✓

≈ 0.004 units ✓

3) [1,2,2,2,4 = 11 marks]

Phil's Phrute Shop sells seedless grapes in bags that have weights that are normally distributed with a mean of 230 grams and a standard deviation of 5 grams.

a) Determine the probability that one of Phil's bags selected at random will weigh exactly 230 grams.

0 ✓

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b) Determine the probability that one of the bags selected at random will weigh between 223 g and 235 g.

0·7605881 ✓✓

c) 5% of the bags of grapes weigh less than w grams. Determine w to the nearest gram.

✓ ✓

$w = 222$ grams to the nearest gram

d) If a customer buys 12 bags of grapes find the probability that all twelve bags weigh between 223 grams and 235 grams.

$(0{\cdot}7605881)^{12} \approx 0{\cdot}03748$ ✓✓

Phil also sells sliced apples in lunch packs. The weights of the lunch packs are also normally distributed. It is known that 5% of the lunch packs weigh less than 153 grams while 12% of the lunch packs weigh more than 210 grams.

e) Determine the mean and standard deviation of the weights of the lunch packs.

$$-1{\cdot}644854 = \frac{\mu - 153}{\sigma}$$

$$1{\cdot}1749868 = \frac{\mu - 210}{\sigma}$$

$\Rightarrow$ $\mu \approx 186{\cdot}248926$ gram ✓✓

$\sigma \approx 20{\cdot}214$ grams ✓✓

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4) [5,3 = 8 marks]

Three functions are given below. Only one of them represents a continuous probability density function X. Identify the probability density function and indicate why the other two are NOT probability density functions and then answer the question that follow for that function that is a pdf.

$f(x)=\dfrac{e}{7x}$ for $[1,7]$ NO: Area $\neq 1$ ✓ ✓

$f(x)=\dfrac{1}{(x+1)^2}$ for $[0,\infty]$ YES ✓

$f(x)=4x^3-4x$ for $[-1,\sqrt{2}]$ NO: Negative for $(0,1)$ ✓ ✓

Find P(0.5 < X < 1.25) exactly.

$$\int_0^{1.25} \frac{1}{(x+1)^2}\,dx$$

$$=\frac{2}{9}$$ ✓✓✓

End of Test

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